bGeigie calibration data and the evolution of Geiger-Müller tube dead time with count rates

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Introduction

The recent article *Assessment of Safecast bGeigie Nano Monitor* by Walsh, Kelleher, and Currivan (WKC) [1] is noteworthy because (i) it confirms that the calibration of the bGeigie Nano for ^{137}Cs is quite accurate at the (gamma) dose rates for which it was designed to operate and agrees well with data from a dedicated, calibrated stationary gamma detector; (ii) it confirms (although this has been long known for the LND 7317 Geiger-Müller tube) that the system readily detects α particles (outside its case), and β particles even with the case closed; and (iii) it provides data in a form which makes the extraction of the 'dead time' of the LND 7317 Geiger-Müller tube straightforward.

The article also provides other useful data for the bGeigie Nano: the angular response as the plane of the Geiger-Müller mica membrane is tilted away from the direction of incident radiation, which must then traverse some or all of the counter wall, and quantitative information about sensitivity to β radiation. The authors selected ^{241}Am (nominally an α emitter) and a mixed ^{90}Sr/^{90}Y source. ^{90}Sr (half life 28.79 years) emits essentially only 0.546 MeV β particles while ^{90}Y (into which it decays) emits 2.280 MeV β, with a 64.6 hour half life. Thus it is possible that the ^{90}Y could be the dominant source of penetrating β particles. Tests with and without the Pelican polycarbonate case (with the detector against the source so that the relatively short range α particles would not have been absorbed by air), together with the assumption that γ rays are unattenuated by the case, provide important information about bGeigie sensitivity to α and β particles. About 36.6% of the counts observed with the case off are attributable to α particles from the ^{241}Am source (the α decay branching ratio is essentially 100%); about 97% of counts detected from a ^{90}Sr/^{90}Y source with the case off are β particles (the β decay branching ratio is essentially 100%).

The authors appear not to have noticed, however, that rather than assessing the accuracy of the bGeigie Nano dose rate as a function of the intensity of incident γ radiation, they were probably conveniently measuring the effects of the Geiger-Müller tube 'dead time', as described below. The fixed, nominal value of the dead time (40 μsec) reasonably accounts for the dose rate range 0-1200 μSv/hr. More careful fits below show the evolution of the effective dead time toward the nominal value as dose rates increase.

Recovery time of Geiger-Müller tubes

Real Geiger-Müller tubes have a 'dead time'–a period after a pulse (produced by an incident particle of ionizing radiation) during which the tube 'recovers' and is hence not sensitive to additional radioactive particles. The LND model 7317 Geiger-Müller tube is described as having a 'minimum dead time' of 40 μsec = 40×10^{-6} seconds. However, when the time between the random arrival of radioactive particles becomes *comparable* to this dead time, a Geiger-Müller counter will miss some particles, will under-count the rate at which particles are arriving, and hence will underestimate the dose rate. There are two common cases often considered to include dead times The simplest model assumes the tube simply 'blanks out' for a fixed dead time after a particle causes a discharge. The second also assumes a a particular value of a parameter still called the the 'dead time', but includes the impact of a *distribution* of dead times determined by the Poisson statistics of radiation counting.

The circuitry itself has its own natural dead time as well. The clock speed of the Arduino FIO used in the bGeigie Nano I believe is 8 MHz. Thus the time for a clock cycle is 0.125 μsec. Because this is much smaller than the quoted dead time, we neglect it—the processor is nimble enough to keep up with whatever nature throws at the Geiger-Müller tube.
A good reference is the book by Knoll [2], *Radiation Detection and Measurement*. His claim is that the reason for the importance of the two models is that real Geiger-Müller times exhibit behavior somewhere between the two.

Before using the data of WKC, it is useful to determine whether 'dead time' corrections have anything to do with what is observed. To this end we use a *fixed* value τ_d of 40 μsec (the 'minimum dead time' quoted by the Geiger-Müller tube manufacturer) in Eq. 4 below to 'correct' the measured dose rates shown in the Table below. The results are shown in Fig. 1.

The measured dose rates are shown as red dots; the green line indicates perfect agreement between measured and specified dose rates, and in blue are shown the corrected dose rates using this fixed dead time. It is clear that while the simple, fixed-τ_d model *overes-*

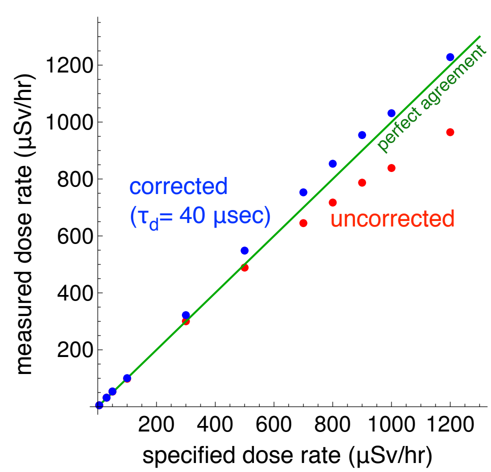

Figure 1: Redrawn data of WKC [1] for bGeigie Nano, red. Blue dots indicate correction for a fixed dead time of 40 μsec and the green line indicates perfect agreement between measured and specified dose rates.

timates actual dose rates, it accounts reasonably for the entire dose range.

Simplest dead time model

Two equations suffice to identify the dead time τ_d in terms of measureable quantitities. First, we identify what we mean by the total dead time during a count:

$$\begin{aligned} T_{act} &= T_{live} + T_{dead} \\ &= T_{live} + N_{meas}\tau_d \end{aligned} \tag{1}$$

thus specifying that the actual elapsed time T_{act} over which counts were made can be decomposed into 'live time' T_{live} during which the tube is available to detect particles and the 'dead time' T_{dead} during which the tube recovers from a discharge. We say the detector is 'dead' but not 'paralyzed': more later.

During the T_{act} we detected N_{meas} pulses, so that the dead time must have been $N_{meas}\ \tau_d$. We can divide both sides of Eq. 1 by T_{act} to find the fraction of time the tube is 'live':

$$\frac{T_{live}}{T_{act}} = 1 - \frac{N_{meas}\tau_d}{T_{act}}. \tag{2}$$

Second, we assume that

$$\frac{T_{live}}{T_{act}} = \frac{N_{meas}}{N_{act}}, \tag{3}$$

that is: the fraction of the actual counts we were able to measure is the same as the ratio of the 'live time' T_{live} to the actual time T_{act}. Now we can deduce what the *actual* number of counts that occurred during the actual time T_{act} must have been. If we plug the value of T_{live}/T_{act} from Eq. 3 into Eq. 2 and solve for N_{act}, we find

$$n_{act} = \frac{n_{meas}}{1 - n_{meas}\tau_d} \tag{4}$$

where (in order to make the denominator dimensionless) we have replaced the count numbers by the corresponding count *rates* (in counts per second) $n_{act} = N_{act}/T_{act}$ and $n_{meas} = N_{meas}/T_{act}$.

Although usually τ_d is considered to be a fixed number, it is worth noting that *if* we have sets of data pairs $\{n_{act}, n_{meas}\}$ we can solve for τ_d for each data pair:

$$\tau_d = \frac{1}{n_{meas}} - \frac{1}{n_{act}} \tag{5}$$

The data of Walsh, Kelleher, and Currivan is precisely of this form.

'Paralyzable' model

The simple model above assumes a *fixed* dead time. However, the actual Poisson distribution of intervals between pulses permits much longer effective dead times as well. The second property of a Poisson distribution in the Appendix is that the probability to find a time interval t between random events is $P(t) = \overline{n}e^{-\overline{n}t}$ where $\overline{n}$ is the *average* rate at which events occur. (If integrated over all times t we get 1.) Thus we can compute the probability of an inter-pulse interval *longer* than a dead time τ_d as

Because the parameter τ is already in use for the dead time, we use a slighty different notation for the Poisson distribution of time intervals between pulses in our Geiger-Müller tube. The *average* rate of pulses can be written as $\overline{n}$ (measured in pulses per second or events per second).

$$P_{>\tau_d} = \int_{\tau_d}^{\infty} \overline{n}e^{-\overline{n}t} = e^{-\overline{n}\tau_d}. \tag{6}$$

If we multiply this by $\overline{n}$ we find the rate at which (on average) events with inter-pulse dead times longer than the parameter τ_d occur: $\overline{n}e^{-\overline{n}\tau_d}$.

For purposes of fitting the WKC data we revert to the notation established for a fixed, simple dead time above: we indicate the rate of *measured* and *actual* pulses by n_{meas} and n_{act}, respectively. Then

$$n_{meas} = n_{act}\, e^{-n_{act}\tau_d}. \tag{7}$$

This expression is conceptually different from the corresponding 'non-paralyzable' result in Eq. 4 because the right-hand side depends now on n_{act} in a transcendental way, forcing us to solve numerically (given n_{meas} and τ_d) for n_{act}. On the other hand, given the data pairs $\{n_{meas}, n_{act}\}$ we can solve for τ_d.

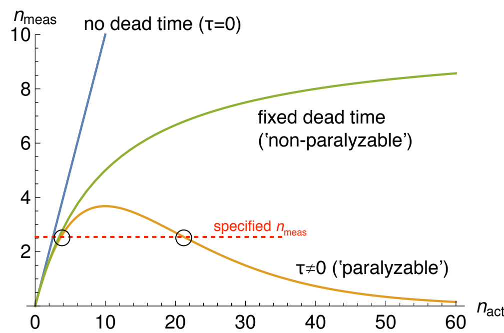

Figure 2: Schematic dependence of measured count rate on actual count rate (note switch of usual ordinate and abscissa) with no dead time (blue), a fixed dead time (green), and a fixed dead time in a paralyzable model. To find the actual count rate in the paralyzable case one must find the intersection of a specified measured rate with the orange curve.

In Fig. 2 we show schematically the qualitative behavior of the curves $f(x) = \{x, x\, e^{-tx}, x/(1+tx)\}$, which are the forms assumed when a counter exhibits (i) no 'dead time', (ii) a non-paralyzable recovery, and (iii) a paralyzable recovery shown in blue, green, and orange, respectively. These forms result when one plots the *measured* dose rate as a function of the *actual* dose rate, not the usual ordinates and abscissas. The horizontal red line specifies an observed measured rate: the intersection of this line with the three curves yields the expected dose rate for each model.

Note that for the orange curve *if* it is the *larger* value of n_{act} that is physically relevant, then we have a situation in which as the actual dose rate *increases* the measured dose rate *decreases*! It is as if the detector is increasingly *paralyzed* as the dose rate increases. The non-monotonic behavior in the paralyzable case is discussed in Knoll and requires careful experimental measurements to identify which intersection is physically relevant. In the case of the WKC data [Fig. 1], however, it is clear that the measured dose rate increases with the actual dose rate, so the 'paralyzable' case behavior will not occur in this dose range.

Extracting dead times from published calibration data

We note that the count rates are linearly proportional to dose rates $\tilde{D}$ measured in μSv/hr. Based on the minimum dead time quoted for the LND 7317, we choose to compute dead times $\tilde{\tau}_d$ measured in μsec. Then $n\tau_d = \alpha\tilde{D}\tilde{\tau}_d$ where

$$\begin{aligned} \alpha &= \frac{334 \text{ counts/min}}{1\mu\text{Sv/hr}} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{10^{-6} \text{ sec}}{1\ \mu\text{sec}} \\ &\simeq 5.5\overline{6} \times 10^{-6}. \end{aligned} \tag{8}$$

In terms of α we may now write

$$\tilde{\tau}_d = \frac{1}{\alpha}\left(\frac{1}{\tilde{D}_{meas}} - \frac{1}{\tilde{D}_{act}}\right) \qquad \text{non-paralyzable} \tag{9}$$

$$\tilde{D}_{meas} = \tilde{D}_{act}\, e^{-\alpha\tilde{D}_{act}\tilde{\tau}_d} \qquad \text{paralyzable} \tag{10}$$

The procedure above results in the values shown in the table below.

exp $\tilde{D}$	meas $\tilde{D}$	corr $\tilde{D}$	simp τ_d	paral τ_{eff}
5	5.2	5.21	–	–
30	31.7	31.93	–	–
50	53.1	53.74	–	–
100	98.3	100.5	31.07	30.80
300	300.3	321.82	–	–
500	488.8	548.50	8.23	8.14
700	645.0	753.17	21.88	21.00
800	717.3	853.64	25.89	24.50
900	787.1	954.36	28.63	26.75
1000	838.6	1031.14	34.57	31.62
1200	964.4	1228.13	36.57	32.72

Table 1: Calibration data for bGeige Nano from Walsh, Kelleher, and Currivan [1]. Exposed, measured, and 'corrected' are dose rates in μSv/hr. 'Dead times' are in μsec. Negative dead times are indicated as -.

In Fig. 3 we show how the extracted dead times for the non-paralyzable case (shown as blue dots) and paralyzable case (shown as green dots) evolve as the specified dose rate increases. It is worth noting that

i For several dose rates the extracted dead time is less than zero, indicating that the data is not precise enough to permit a robust solution; this tends to occur at low dose rates where the measured and actual dose rates are very close in value. (The 100 μSv/hr point is an outlier.) Above about 300 μSv/hr, however, the extracted dead time values increase smoothly with dose rate (apart from some scatter in the measured data).

ii At low dose rates the extracted dead times are close in value. (It is widely known that the non-paralyzable and paralyzable results

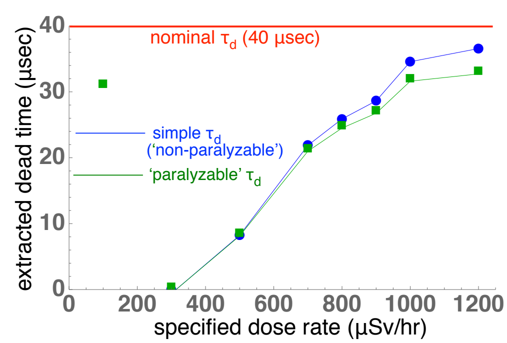

Figure 3: Evolution of the extracted dead time with specified dose rate in a non-paralyzable model (blue), in the paralyzable model (green). The fixed, nominal 'minimum dead time' for the Geiger-Müller tube is shown in red.

coincide to lowest order in τ, which can be taken as zero at very low dose rates.)

iii Both sets of extracted dead times appear to be asymptotically approach (distinct) constants not far from the nominal 40μsec quoted dead time. This is consistent with having identified Geiger-Müller tube dead times as the principal origin of the systematic differences between specified and measured dose rates.

iv As noted above, any dependence of the dead time on dose rates is an indication that a fixed dead time fails to account for the kinetics of pulse relaxation. The extracted dead times indicate that (in effect) relaxation is faster at low dose rates than at high.

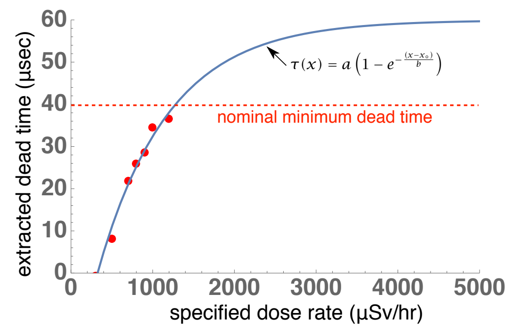

Figure 4: Fit to the dose rate dependence of the simple Geiger-Müller dead time extracted from WKC data.

Using WKC data to construct a correction curve

We use a parameterization of the evolution of the non-paralyzable (simple) dead time with specified dose rate of the convenient form

$$\tau(x) = a\left(1 - e^{-\frac{(x-x_\circ)}{b}}\right) \tag{11}$$

for dose rate x. (Only simple operations and functions have been chosen.)

This form permits ready identification of the dose rate $x_\circ$ at which τ_x first becomes positive (an acknowledgement of small differences and hence numerical noise between specified and measured rates at low dose rates), the characteristic dose rate scale b over which the dead time 'turns on' and increases, and the saturation (high dose rate) value of the dead time. For x in μSv/hr, we find best fit parameters $a = 59.95$, $x_0 = 324.45$, $b = 867.99$ with a fit "R squared" of 0.9963 (a good fit). Standard errors, in percent, for these fit parameters are about 26%, 8%, and 41%, respectively. An extrapolation to very high dose rates is shown in Fig. 4. Note that 5000 μSv/hr corresponds to more than 1.6 million counts per minute, so these high dose values are physically irrelevant and should not be trusted from an extrapolation from much lower dose rates.

A more useful application of a parameterized dependence of the dead time on dose rate is to construct a 'correction curve' from which the *actual* dose rate can be predicted from the bGeigie Nano-measured dose rate. This is shown in Fig. 5; WKC data points are shown as blue dots. Differences between the corrected and incident dose rates are no more than 4% at moderate to high dose rates. This figure should be compared with the original curves in Fig. 1.

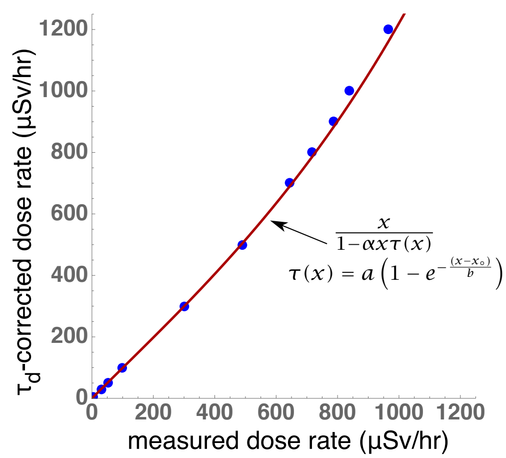

Figure 5: Correction curve including dose-dependence of the dead time extracted from WKC data.

Takeaway points

- What appear to be errors of calibration for the bGeigie Nano are in fact the onset of dead time effects for the LND 7317 Geiger-Müller tube beyond about a dose rate of 300 μSv/hr (or 100,000 counts per minute, using the Safecast ^{137}Cs calibration of 334 CPM = 1 μSv/hr).
- For very high count rates the non-paralyzable dead time appears to saturate at an extrapolated value of about 60 μsec.
- From the viewpoint of the Safecast project
  1. The data of Walsh, Kelleher, and Currivan confirm very good calibration of the bGeigie Nano at low to moderate dose rates.
  2. There is *already* support for dead-time correction (of the non-paralyzable form) for the bGeigie Nanon: the GitHub bGeigie Nano codereads

     ```
     c_p_m = (unsigned long)((float)c_p_m/(1-(((float)c_p_m*1.8833e-6))));
     ```

     which (since the count rate is presumably in CPM) means an assumed dead time of 113 μsec: equating `1.8833e-6` with

     $$\tau(\text{in } \mu\text{sec}) \times \frac{10^{-6}\text{ sec}}{\mu\text{sec}} \times \frac{1\text{ minute}}{60\text{ sec}}$$

     The accompanying comment
     `// deadtime compensation (medcom international)`
     implies this is a long-standing value.
  3. Since curent bGeigie Nano specifications indicate a maximum operating value of 1000 μSv/hr, it makes some sense to evaluate the dead time at this value, where is is 34.6 μsec (see Table 1).
- Questions for Safecast:
  Why this value of dead time (113 μsec)? It is much larger than the dead time extracted directly from the WKC data and than the asymptotic large-dose limit value extracted above.
  Precisely when is dead time correction turned on? For which displays? For 'xGeigie' *and* 'bGeigie" modes? In the LOG file? Is it applied during post-processing of the LOG file by the API?
  If it was automatically turned on, why did the authors find a 'calibration curve' that looked exactly like the raw data (uncorrected for dead time)? They were actually filming the display while the bGN was in 'bGeigie' (logging) mode, so they had both a LOG file and the filmed record. Thus both the display and the LOG file would appear to show un-corrected data. Did the authors turn off the alarm dose rate (150 CPM).

References

[1] Joshua Walsh, Kevin Kelleher, and Lorraine Currivan. "Assessment of Safecast bGeigie Nano Monitor". In: *Radiation Environment and Medicine* 8.1 (2019), pp. 1–8.

[2] Glenn F. Knoll. *Radiation Detection and Measurement*. Third. John Wiley & Sons, 2000. ISBN: 0-471-07338-5.

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Appendix: Statistics of radioactive decay

For our purposes we can summarize Poisson statistics via the properties

1. *Decay probability in a small time interval:* The probability a given nucleus decays in a differential time dt is $1/\tau\ dt$, where $1/\tau$ is the *average* decay *rate*. *Warning:* the parameter τ is conventional notation and has nothing to do with a 'dead time'. Its physical significance is entirely that the *average* rate at which decays occur is $1/\tau$, with units decays per second. Thus we could equally well write the average decay rate as $\overline{n}$.

2. *Survival time before decay:* Using the result above the probability P_s (s indicates survives) the nucleus 'survives' to a time $t + dt$ without decay can be written

$$P_s(t+dt) = P_s(t) \times \left(1 - \frac{dt}{\tau}\right),$$

where $1 - \frac{dt}{\tau}$ is the probability that the nucleus does *not* decay (that is, survives) during a time interval dt. Expanding out for small dt,

$$\begin{aligned} P_s(t+dt) &= P_s(t) - P_s(t)\frac{dt}{\tau} \\ &\simeq P_s(t) + \frac{dP_s}{dt}dt, \end{aligned}$$

so that (since the term $P_s(t)$ is common to both sides)

$$\frac{dP_s(t)}{dt} \times dt = -P_s(t)\frac{dt}{\tau},$$

whose solution is (assuming we start watching at time $t = 0$)

$$P_s(t) = e^{-\frac{t}{\tau}} = 2^{-t/t_{\frac{1}{2}}}$$

where evidently $t_{\frac{1}{2}} = \tau \ln 2$. This expression also holds for the time interval between *events* which obey Poisson statistics, such as the time between pulses in a Geiger-Müller detector due to random incoming radiation.

As you know, the survival of a large collection of identical radioactive nuclei is described by their 'half life': the time it takes for half of this collection to have decayed. Thus we have related the decay behavior of an average nucleus to the half life of a sample of identical nuclei.