Why you should care about radon

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1 *An introduction to radon*

The easiest way to substantially reduce your exposure to ionizing radiation is to make sure your basement does not have high levels of radon. As noted by Simms *al.* [1],

> The current particle radiation dose rate to lungs from residential radon in Canada is 4.08 mSv/y from 108.2 Bq/m^3, with 23.4% receiving 100–2655 mSv doses that are known to elevate human cancer risk. Notably, residences built in the twenty-first century are occupied by significantly younger people experiencing greater radiation dose rates from radon (mean age of 46 at 5.01 mSv/y), relative to older groups more likely to occupy twentieth century-built properties (mean age of 53 at 3.45–4.22 mSv/y). Newer, higher radon-containing properties are also more likely to have minors, pregnant women and an overall higher number of occupants living there full time. As younger age-of-exposure to radon equates to greater lifetime lung cancer risk, these data reveal a worst case scenario of exposure bias. This is of concern as, if it continues, it forecasts serious future increases in radon-induced lung cancer in younger people.

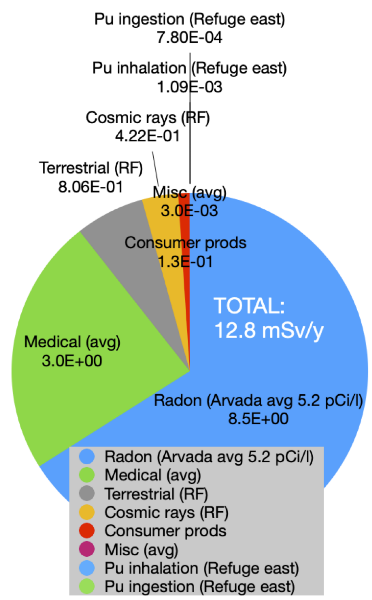

Figure 1: The majority of annual radiation dose in Arvada comes from unmitigated radon.

Radon is a chemically inert and heavier-than-air gas, so you are constantly breathing it in, in small amounts. One of its isotopes, ^{222}Rn, is part of the radioactive decay chain of ^{238}U and another (^{220}Rn, 'thoron') is part of the decay chain of ^{232}Th; The uranium and thorium isotopes are 'parents' of these decay chains. There is plenty of the parent isotopes in Colorado soil and both radon isotopes are radioactive. The risk of radon is *not* due to the gas itself (since its 3.82 day half-life is much longer than the time between breaths, so essentially all radon breathed in is breathed back out). Instead, the risk comes from the radioactive 'daughter' atoms into which radon decays (by alpha and beta emission). Alpha and beta decay leave the daughter atoms in a charged state so they are attracted to charged objects such as dust particles. It is these aerosols which can be trapped in the lungs long enough to do biological damage as further decays occur. The net effect is *still* that the biological damage remains determined by the concentration of radon in household air.

These are typically very small and are described as aerosols, rather than 'hot particles', much larger extended objects.

Only very recently [2, 3] has epidemiological data become available for the relative doses of radon and thoron (which reflect the soil concentrations of the uranium and thorium parents, which can vary widely from place to place). Ref. [2] (for people living around Qingyang City in Gansu Province in *the* Loess Plateau) used the International Commission on Radiological Protecion (ICRP) 'dose conversion factors" 9 nSv per (Bq/m^3 hour) for radon and progeny and 40 nSv per (Bq/m^3 hour) for thoron and progeny. This indicates that strictly speaking thoron should *not* be ignored in regions where the soil has thorium concentrations near that of uranium.

Both radon isotopes diffuse through soil as uranium and thorium atoms decay. Because the concrete slab in your basement is rigid, thick, and smooth Rn tends to diffuse into your basement through cracks or seams. For this reason a careful radon mitigation contractor will seal the joint between the concrete slab and the concrete footers around the perimeter of the basement. Most mitigation systems consist of a pump that 'sucks' on a cavity at the bottom of the house below the slab. In the case of my house, an artificial hole was drilled through the slab into dirt and enlarged a little, so that radon would accumulate there if it weren't constantly being pumped up a PVC pipe. (Sometimes the cavity is the sump of your house.)

Diffusion doesn't care if an atom is heavier than air–it is driven by differences in concentration from place to place, the 'gradient' of the Rn concentration in this case. Including gravity would probably cause a slight pile up of radon near the basement floor at the expense of more complicated math.

I believe that Jefferson County code requires radon abatement system pipes to vent to the roofline of the house, so that Rn does not accumulate anywhere near where people are breathing.

2 *More for the curious*

The concentration of radon atoms in air obeys the 'diffusion equation', which relates the rate (in time) at which the atoms 'diffuse' to how rapidly the concentration changes with distance. Like every 'partial differential equation', this can be solved only with 'initial conditions' (specifying the concentration of radon at some nominal initial time) and with a 'boundary condition' about the radon concentration on a boundary.

While the diffusion equation can certainly be solved numerically for realistic descriptions of real basements, it is simpler as an illustration to assume that the basement is much larger in transverse

To be precise, the boundary condition here reflects the presence of a constant trickle of radon through the slab and into the bottom of the basement. Is it really *constant*? No, it could change with rainfall or a change in the temperature of the soil or air or the barometric pressure, but these are unnecessary complications if we'e mostly interested in the basic behavior.

dimensions than it is tall. In fact, we'll assume that the floor is an *infinite plane* for simplicity. Because radon is seeping in at the bottom of the basement, the concentration of radon will depend on the distance above the floor and on time. Since ^{222}Rn is radioactive, we *also* need to include its half-life.

Theoretical physicists just love to use 'dimensionless variables' (variables without units) because they allow you to immensely extend the validity of a calculation if you know what you're doing. Following is a problem based on this situation.

2.1 *Details of solution of diffusion equation*

Radon diffusion [*Butkov* Problem 8.14, with hints]
A radioactive gas diffuses into the atmosphere from the soil at a rate χ [in units of, for example, μg/(cm^2-sec)]; see Fig. 2.

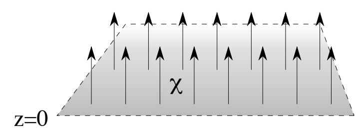

Figure 2: Diffusion of radioactive species into a semi-infinite space

(The integrals in this problem cannot be done by *Mathematica* without help, and the problem itself is extremely relevant to Colorado residents, since radon in basements is a ubiquitous issue.)

We assume the ground and the atmosphere are semi-infinite media with $z = 0$ as the boundary and that the density $\rho(z,t)$ of gas [in similar units, say μg/cm^3] obeys the diffusion equation (see below). We take the boundary and initial conditions to be

$$\rho(\infty,t) < \infty \quad (1)$$

$$\rho(z,0) = 0 \text{ for } z \geq 0; \quad (2)$$

one more follows below.

The ordinary continuity equation for the particle diffusion current $\mathbf{j}_{\text{dif}}$ is

$$\nabla \cdot \mathbf{j}_{\text{dif}} + \frac{\partial \rho}{\partial t} = 0. \quad (3)$$

We assume, as usual, that $\mathbf{j}_{\text{dif}} = -D\boldsymbol{\nabla}\rho$ where D is the diffusion coefficient of the species in air.

(a) For a radioactive species, argue that the diffusion equation must become

$$\frac{\partial \rho(\mathbf{r},t)}{\partial t} = D\nabla^2 \rho(\mathbf{r},t) - \lambda \rho(\mathbf{r},t). \quad (4)$$

Relate the constant λ to an observable property of the radioactive species (*hint:* think about the situation if there were no spatial variations). Also, show that

$$\left.\frac{\partial \rho}{\partial z}\right|_{z=0} = -\frac{\chi}{D}, \quad (5)$$

is an additional boundary condition.

Warning: If we're considering a semi-infinite region, we can't be casual about the 'surface terms' we get when (cosine) transforming $\frac{\partial^2 \rho}{\partial z^2}$. You will find three additional terms which depend on $\rho(z \to \infty)$ and $\frac{\partial \rho}{\partial z}$ at $z = 0$. Why didn't we use a sin transform?

(b) The symmetry of the problem implies that ρ can vary only in the z direction. Solve the problem using a Fourier cosine transform to show that

$$\rho(z,t) = \frac{2\chi}{\pi} \int_0^\infty dk \; \cos kz \; \frac{1 - e^{-(Dk^2+\lambda)t}}{Dk^2 + \lambda} \quad (6)$$

(c) Solve the *same* problem using Laplace transforms to show that

$$\rho(z,t) = \frac{\chi}{\sqrt{D\pi}} \int_0^t d\tau \; \frac{1}{\sqrt{\tau}} e^{-\lambda \tau} e^{-\frac{z^2}{4D\tau}}. \quad (7)$$

Hints: (i) the inverse Laplace transform of $\frac{1}{\sqrt{s}}\exp[-\beta\sqrt{s}]$—long broken in *Mathematica*—now works, but it cannot evaluate what you need. The correct answer should be

You could do this!

$$\mathcal{L}^{-1}\left[\frac{\exp\left(-\beta\sqrt{\lambda+s}\right)}{\sqrt{\lambda+s}}\right] = \frac{e^{-\lambda t-\frac{\beta^2}{4t}}}{\sqrt{\pi t}} \quad (8)$$

(ii) you may also find useful the factoid that $\mathcal{L}^{-1}[F(s+\lambda)] = e^{-\lambda t}F(t)$; (iii) Use the convolution theorem.

(d) Show that these two apparently different solutions are equivalent by reducing one to the other. *Hints:* Use the fact that

$$\int_0^t dt' e^{-(Dk^2+\lambda)t'} \quad (9)$$

is related to our result in part (b) and do the k integral.

(e) *Results:* Starting with Eq. (7), (i) define new dimensionless variables

$$\tau \equiv \lambda t \; [= t/\lambda^{-1}] \quad (10)$$

$$x \equiv z\sqrt{\frac{\lambda}{4D}}, \quad (11)$$

then (ii) make the change of variables $\tau \to u^2$. Use *Mathematica* to evaluate the *indefinite* integral that results. Determine the value of the integral at the lower limit. Show that

$$\rho(x,\tau) = \sqrt{\pi}\exp(-2x) - \frac{\sqrt{\pi}}{2}\left[\exp(-2x)\,\mathrm{erfc}\frac{\tau-x}{\sqrt{\tau}} + \exp(2x)\,\mathrm{erfc}\frac{\tau+x}{\sqrt{\tau}}\right]. \quad (12)$$

(f) Plot this result with *Mathematica* as a function of x and τ and discuss the physics.

2.2 *Results of diffusion solution*

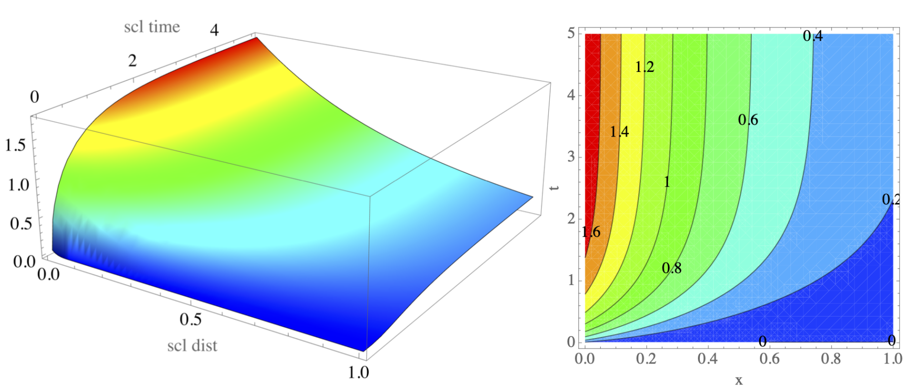

Figure 3: Two ways of visualizing the solution.

2.3 *Extracting half life and diffusion coefficient for radon in air from real data*

A nice, straightforward experiment [4] provided figures for the time evolution of the radon concentration at $z = 0$ (the 'floor' in their experiment), and for the dependence on height z) in the steady state ($t \to \infty$), which I digitized and fit. The limiting forms of the scaled radon concentration are

$$\rho(z,t) \simeq \begin{cases} \text{erf}\left[\sqrt{\log 2}\,\frac{t}{t_{\frac{1}{2}}}\right], & z \to 0 \\ \sqrt{\pi} e^{-z\sqrt{\frac{\log 2}{D_0\, t_{\frac{1}{2}}}}}, & \tau \to \infty \end{cases} \tag{13}$$

Thus the half-life can be identified simply from the behavior at $z = 0$ and the diffusion coefficient from the exponential in the steady-state z dependence. (In principle the diffusion equation should be solved in cylindrical coordinates with additional boundary conditions $\rho(r = R, z) = 0$ reflecting the fact that radon cannot diffuse through the curved wall of radius 4.5 cm of the cylinder corresponding to the experimental geometry.)

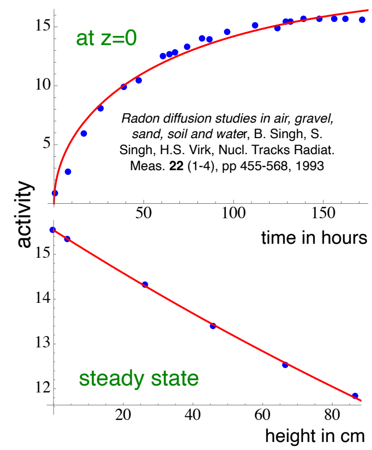

Figure 4: Compare with dependence shown in the rectangular geometry discussed above. The small-t deviations are probably due to the fact that a small-diameter cylinder (rather than a very large transverse size) was used in the experiments.

source	$t_{\frac{1}{2}}$ (days)	D_0 (cm^2/sec)
article	–	0.211
fits here	4.14	0.191
nominal	3.82	0.11

Table 1: Values of ^{222}Rn half life and diffusion coefficient in air. Nominal diffusion coefficient from here or here.

3 *Results in your basement*

Using the nominal values above, we find the following spatial and time dependence after the 'leak' is first turned on. Just as the dependence on time directly at the source looks like a 'capacitor charging' curve, so if the leak is stopped we expect a capacitor discharging curve as the radon concentration decays back to zero with the same characteristic time.

4 *Measured radon values before and during mitigation*

Why mitigate? On the last pages is data acquired using a radon meter. The meter was about 1m above the floor. It is tempting to claim that the time for clearance is related to the actual half-life of ^{222}Rn, but this is just plausible, not definitive.

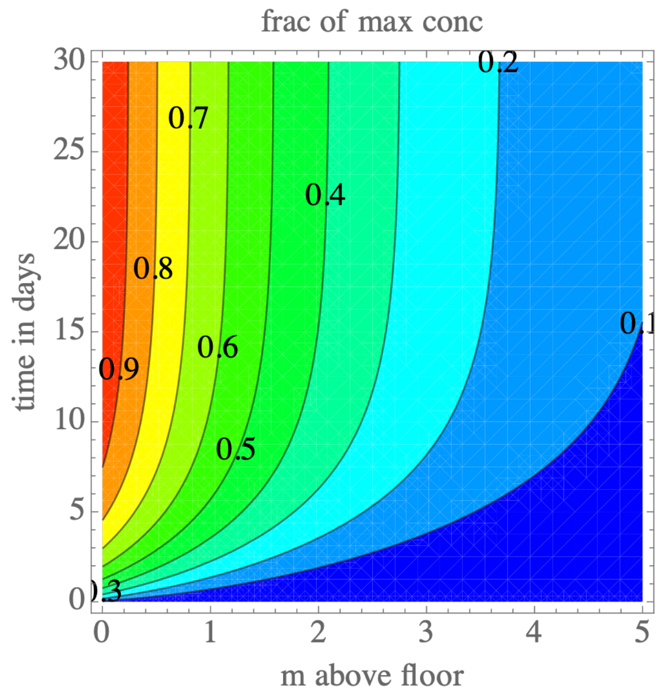

Figure 5: Using nominal values for ^{222}Rn half life and diffusion coefficient in air.

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[4] Baljinder Singh, Surinder Singh, and H. S. Virk. "Radon diffusion studies in air, gravel, sand, soil and water". In: *Nuclear*

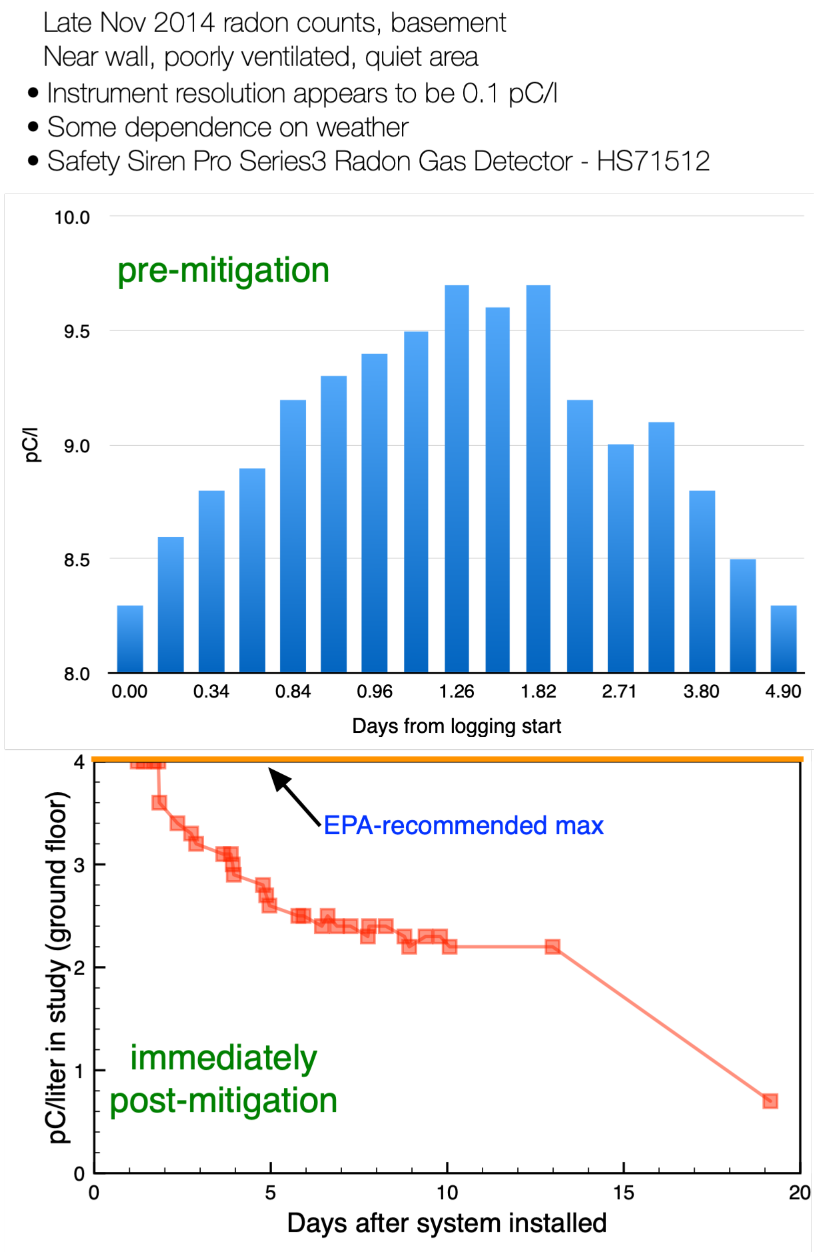

Figure 6: Data acquired with a radon meter (pre-mitigation, at top, acquired in winter when radon levels are typically highest. Bottom: Decay of initial radon concentration after mitigation systemm installed.

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